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2001 J. Phys.: Condens. Matter 13 L885

(http://iopscience.iop.org/0953-8984/13/44/101)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **13** (2001) L885–L889 PII: S0953-8984(01)27936-3

LETTER TO THE EDITOR

Spatial localization of light flux in an array of nonlinear optical waveguides

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Received 16 August 2001
Published 19 October 2001
Online at stacks.iop.org/JPhysCM/13/L885

Abstract

The stationary localized states of light fluxes propagating in periodic systems of plane-parallel nonlinear waveguides are investigated. It is shown that the problem is reducible to that of a model of connected anharmonic oscillators. All of the parameters of such an oscillator model are found exactly from the microscopic description of the problem. The solution for an optical beam localized perpendicularly to the direction of its propagation is analytically derived.

Investigations of the propagation and the character of localization of nonlinear waves in modulated systems have always been a focus of attention in nonlinear wave dynamics. Theoretical and experimental investigations of spatial localization of high-power light beams have received considerable attention in recent years. Localization of light flux (perpendicularly to the direction of its propagation via the nonlinear Kerr effect) in a nonlinear homogeneous optical medium was discovered by Chiao *et al* [1]. The theory of this phenomenon was described in reference [2]. On the other hand, such transverse localization of light flux is possible in a linear optical medium near a planar waveguide [3]. The spatial localization of a nonlinear optical beam in a few neighbouring waveguides in a medium with Kerr nonlinearity was investigated theoretically by Aceves *et al* [4]. Numerical simulations proving this result within the framework of the discrete nonlinear Schrödinger equation (DNLSE) for the field amplitudes in waveguides were reported; however, the interaction of the optical waveguides was described by a phenomenological parameter and the origin of nonlinearity in the equation was not discussed in [4]. Later such 'superlocalization' of light flux was observed experimentally by Eisenberg *et al* [5] and the results were compared with a phenomenological discrete model in reference [4]. In our previous paper [6] we described the propagation of nonlinear wave flux along two coupled plane-parallel waveguides in an anharmonic medium. Assuming that the waveguides and environment are both nonlinear and differ in linear refractive index, the degree

0953-8984/01/440885+05$30.00

of interaction of light beams in waveguides was analytically derived and the discrete nonlinear dynamical equations describing the field amplitudes in the waveguides were obtained. We demonstrated the possibility of localization of nonlinear wave flux in one waveguide.

The goal of the present letter is to investigate the localization of nonlinear stationary waves propagating along a system of identical plane-parallel nonlinear optical waveguides. Note that investigations of nonlinear properties of waveguide arrays are usually performed by using discrete models for the wave amplitudes in waveguides [4, 5, 7–10] which are typically described by phenomenological equations with arbitrary parameters. We take into account the nonlinear Kerr terms only in the waveguides (assuming that the width of the waveguides is much smaller than the distance between adjacent ones) because of the smallness of the average field amplitude over large regions for weak waveguide coupling. Note that in systems with quadratic nonlinearity, the nonlinear terms have to be considered only within interfaces [11]. We can also describe the array of optical waveguides in vacuum.

For the system proposed, the equation for the envelope of the nonlinear monochromatic wave $E(z, t)$ (the z-axis is perpendicular to the plane of the parallel waveguides) propagating along the waveguides is an ordinary nonlinear Schrödinger equation (NLSE):

$$\mathrm{i}\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial z^2} = -\lambda \sum_{j=-\infty}^{+\infty} \delta(z - 2aj)|E|^2 E \tag{1}$$

where the parameter $\lambda > 0$ for the waveguides.

For one waveguide the equation

$$\mathrm{i}\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial z^2} = -\lambda\delta(z)|E|^2 E \tag{2}$$

has the following solution for a stationary localized beam:

$$E = E_0 \exp(-\varepsilon|z| - \mathrm{i}\omega t) \tag{3}$$

where

$$\varepsilon = \sqrt{-\omega} \qquad \text{and} \qquad E_0 = \sqrt{\frac{2}{\lambda}}\sqrt{\varepsilon}.$$

Hence we come to a dependence of a parameter of a wave ω on the field amplitude in the waveguide that has the same form as the dependence of the frequency of an anharmonic oscillator on the amplitude of its oscillations:

$$\omega = -\frac{\lambda^2}{4} E_0^4. \tag{4}$$

Note that if we introduce the total ‘intensity’ of the optical flux in the form

$$W = \int_{-\infty}^{+\infty} |E|^2 \, \mathrm{d}z \tag{5}$$

and its total energy as

$$U = \int_{-\infty}^{+\infty} \mathrm{d}z \left\{ \left|\frac{\partial E}{\partial z}\right|^2 - \frac{\lambda}{2}\delta(z)|E|^4 \right\} \tag{6}$$

then neither of the parameters of our model depend on the frequency ω: $W = 2/\lambda$, $U = 0$. But this property is not universal. Taking into account the nonlinearity in the regions outside the waveguide (adding the term $2|E|^2 E$ into the left-hand side of equation (2)), we obtain

$$W = \frac{2}{\lambda}\left[1 + \varepsilon\lambda - \sqrt{1 + \varepsilon^2\lambda^2}\right].$$

If we consider only a linear refractive index in the waveguide (when the right-hand side of equation (2) is equal to $-\lambda\delta(z)E$), then we shall obtain the following dependence [11]: $W = 2(\varepsilon - \lambda/2)$.

Let us consider the system of two plane-parallel waveguides at the positions $z = \mp a$. In this case the problem is reduced to the linear wave equation

$$\mathrm{i}\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial z^2} = 0 \tag{7}$$

with the following boundary conditions at the waveguide positions:

$$E\big|_{\mp a-0} = E\big|_{\mp a+0} \tag{8}$$

$$\left.\frac{\partial E}{\partial z}\right|_{\mp a+0} - \left.\frac{\partial E}{\partial z}\right|_{\mp a-0} = -\lambda(|E|^2 E)\big|_{\mp a}. \tag{9}$$

The light flux localized in the waveguide system is described by the solution in the following form:

$$E_{\alpha,\beta} = A_{\alpha,\beta}\mathrm{e}^{\pm\varepsilon z - \mathrm{i}\omega t} \tag{10}$$

$$E_\gamma = (B\mathrm{e}^{-\varepsilon z} + C\mathrm{e}^{\varepsilon z})\mathrm{e}^{-\mathrm{i}\omega t} \tag{11}$$

in the regions α ($z < -a$), β ($z > a$), and γ ($-a < z < a$). Using the solution (10), (11) we can rewrite the boundary conditions (8) and (9) as

$$\varepsilon^2 E_i - \frac{\lambda\varepsilon}{2}(1 + \mathrm{e}^{-2\varepsilon a})E_i^3 + \frac{\varepsilon^2}{\mathrm{e}^{2\varepsilon a} - 1}(E_i - E_j) = 0 \tag{12}$$

where $i, j = 1, 2$, $i \neq j$, and $E_{1,2} = E(z = \mp a)$ are the field amplitudes in the waveguides. For weak coupling of the waveguides ($\varepsilon a \gg 1$) (large distance between them or strong localization of the wave in the waveguides), equations (12) describe the stationary oscillations of two weakly coupled anharmonic oscillators:

$$\varepsilon^2 E_i - \frac{\lambda\varepsilon}{2}E_i^3 + \varepsilon^2\mathrm{e}^{-2\varepsilon a}(E_i - E_j) = 0 \tag{13}$$

with a binding energy

$$U_{int} = \frac{1}{2}\varepsilon^2\mathrm{e}^{-2\varepsilon a}\left|E_i - E_j\right|^2. \tag{14}$$

It is an important feature of the result obtained that the oscillator's binding energy depends not only on the parameters of our system but also on the parameter ω of our solution. Therefore, as for the modelling of the plane-parallel waveguide array by a chain of oscillators, the interaction is, in fact, a function of the frequency of a monochromatic wave propagating in the system. (In papers [4, 5] an effective constant of interaction was involved.)

The system of equations (12) allows three types of possible stationary state: the symmetric state (S) with equal fluxes in two waveguides:

$$E_1 = E_2 = \sqrt{\frac{2\varepsilon}{\lambda}}(1 + \mathrm{e}^{-2\varepsilon a})^{-1/2} \qquad \text{(S)} \tag{15}$$

and the antisymmetric state (A) with equal fluxes but with opposite phases of the field in the waveguides:

$$E_1 = -E_2 = \sqrt{\frac{2\varepsilon}{\lambda}}(1 - \mathrm{e}^{-2\varepsilon a})^{-1/2} \qquad \text{(A)} \tag{16}$$

and also the inhomogeneous state (N) with equal phases but unequal fluxes in two waveguides:

$$E_{1,2} = \sqrt{\frac{\varepsilon}{\lambda}}\left(\frac{1 \pm \sqrt{1 - 4\mathrm{e}^{-4\varepsilon a}}}{1 - \mathrm{e}^{-4\varepsilon a}}\right)^{1/2} \qquad \text{(N)}. \tag{17}$$

For the solutions obtained, the total intensity W takes the form

$$W_S = \frac{4}{\lambda}\frac{1+(2\varepsilon a+1)e^{-2\varepsilon a}}{(1+e^{-2\varepsilon a})^3}$$
$$W_A = \frac{4}{\lambda}\frac{1-(2\varepsilon a+1)e^{-2\varepsilon a}}{(1-e^{-2\varepsilon a})^3} \qquad (18)$$
$$W_N = \frac{2}{\lambda}\frac{1-3e^{-4\varepsilon a}+2(2\varepsilon a+1)e^{-8\varepsilon a}}{(1-e^{-4\varepsilon a})^3}.$$

The inhomogeneous state (N) is split off from the symmetric state (S) via a bifurcation. The value at bifurcation of the total intensity of the optical flux is equal to

$$W_b = \frac{16}{27}(3+\ln 2)\frac{1}{\lambda} \qquad \left(\varepsilon_b = \ln\sqrt{2}\frac{1}{a}\right).$$

The dependences (18), in the form $\omega a^2 = f(\lambda W)$ for all the types of stationary state, are shown in figure 1. The solid curves represent stable regions and the thick dashed curve corresponds to the region of instability for a symmetric state.

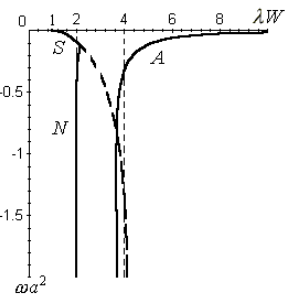

Figure 1. The dependence $\omega a^2 = f(\lambda W)$ for the in-phase symmetric mode (S), the inhomogeneous state (N), and the antiphase mode (A).

As follows from the theory of connected anharmonic oscillators (see for instance [12]), the number of stationary modes in the system increases as the number of anharmonic oscillators grows. At a fixed frequency, the spatially localized state of in-phase oscillating particles (the 'discrete breather') has the minimal energy. Thus a point of bifurcation (the threshold value of total energy at which this localized state is split off from the homogeneous in-phase state) is moved into the region with small energies as the number of connected oscillators grows. In an infinite chain, the 'breather' state can exist at any small energies.

In our case the infinite system of the plane-parallel waveguides can be reduced to a system of connected anharmonic oscillators, just as the above pair of waveguides was described by a model of two coupled oscillators. Returning to the initial equation (1) and to the boundary conditions (8) and (9) for all the waveguides, we obtain the stationary solution in the region between waveguides j and $j+1$ in the following form:

$$E_{j,j+1} = A_j^{(+)}\exp\{-\varepsilon(z-2ja)-i\omega t\} + A_{j+1}^{(-)}\exp\{\varepsilon(z-2ja-2a)-i\omega t\}. \qquad (19)$$

Then for the field amplitudes in the waveguides $E_j = A_j^{(+)} + A_{j+1}^{(-)}e^{-2\varepsilon a}$, it is easy to obtain the infinite system of connected nonlinear algebraic equations:

$$\varepsilon^2 E_j - \frac{\lambda\varepsilon}{2}\coth(\varepsilon a)E_j^3 + \left(\frac{\varepsilon}{2\sinh(\varepsilon a)}\right)^2(2E_j - E_{j+1} - E_{j-1}) = 0. \qquad (20)$$

These equations formally coincide with the equations describing a chain of connected oscillators and with the equations used in the papers [4, 5]. But in equations (20) all of the coefficients not only are functions of the characteristics of the medium but also depend substantially on the parameter of the wave ω.

The spatially localized solutions of systems like (20) were investigated earlier (see [13] and references therein). In our case of weak effective connection between waveguides ($\varepsilon a \gg 1$) the light flux is localized mainly in one waveguide. Such solutions were investigated in detail in the paper [11]. In the opposite limit case ($\varepsilon a \ll 1$), the field amplitudes in neighbouring waveguides differ weakly and the light flux is localized in a large number of waveguides. So, we have the so-called 'supersoliton'. In this case equations (20) are replaced by differential equations having obvious soliton solutions:

$$E_j = \sqrt{\frac{4a}{\lambda}} \frac{\varepsilon}{\cosh(2\varepsilon j a)}.$$

Thus one can easily obtain that the light flux is localized in $N \sim 1/(\varepsilon a) \gg 1$ waveguides. Note that in the limit case $\varepsilon \to 0$ the total intensity of optical flux in the system with one waveguide is equal to $W_1(\varepsilon \to 0) = 2/\lambda$. In the system with two waveguides the limit value of the intensity becomes $W_2(\varepsilon \to 0) = 1/\lambda$. In an infinite system of plane-parallel waveguides the total intensity of wave flux, as follows from the solutions of system (20), is equal to $W = 8\varepsilon a/\lambda$ and vanishes in the limit case $\varepsilon \to 0$.

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